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VOLUMETRIC RATE OF A TURBULENT NEWTONIAN FLUID FLOW IN A CYLINDRICAL CHANNEL

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A new formula to calculate the volumetric rate of a turbulent Newtonian fluid flow is suggested. Experimental verification of the validity of the formula for fluids of different physicochemical nature passed through a cylindrical channel of a laboratory rig has been made.

Introduction. During flow of a fluid in a cylindrical channel two sharply different regimes can be realized: a laminar or a turbulent one. But irrespective of the regime of flow the linear velocity of particles changes from maximum on the axis to zero on the tube wall, where the condition of "mechanical adherence" holds. In a laminar flow a parabolic distribution of velocities over the channel cross section is observed; it is described by the equation

$$U = \frac{\Delta P}{4L\eta}\left(R_{\mathrm{w}}^2 - R^2\right) \quad \text{or} \quad U = \frac{\tau_{\mathrm{w}}}{\eta}\, y\left(1 - \frac{y}{2R_{\mathrm{w}}}\right),$$

where $R = R_{\mathrm{w}} - y$.

The amount of the fluid flowing in a laminar regime through the tube cross section per unit time (volumetric rate of flow) is equal to the volume of the paraboloid of revolution (the Poiseuille formula) and can be calculated after $Q = \int UdS$ is integrated.

Statement of the Problem. The complex picture of a turbulent flow has no rigorous theoretical justification up to now. At the present time the most widespread theory is the Prandtl semi-empirical theory [1] based on a simplified model of a two-layer flow in a cylindrical channel.

According to this schematic model, a turbulent flow in a tube consists of a turbulent core in which the main mass of the fluid moves with a logarithmic velocity profile and the thinnest wall zone in which the laminar flow laws are valid. Within the framework of this theory an equation describing the experimentally observed logarithmic profile of velocities with a turbulent mode of flow in a tube has been obtained;

$$U = \frac{u_*}{\xi} \ln\left(\frac{u_*}{\nu} y\right) + C\,, \tag{1}$$

where $u_* = \sqrt{\tau_{\mathrm{w}}/\rho}$; $\nu = \eta/\rho$.

Even though Eq. (1) provides valuable information on the distribution of velocities in the turbulent flow core, it has substantial limitations. First, it cannot be used for the wall region, where it loses its physical meaning, since when $y \to 0$, the inequality $yu_*/\nu < 1$ holds, and according to Eq. (1) the linear velocity becomes negative. Second, the function $U = (u_*/\xi)\ \ln\ (yu_*/\nu)$ is not found at $y = 0$ and, consequently, it cannot be integrated ($Q = \int UdS$) within the limits from $y_1 = 0$ to $y_2 = R_{\mathrm{w}}$ to obtain a formula for calculation of the volumetric rate of flow, as was done when deriving the Poiseuille equation. Therefore, in carrying out technological calculations of the volumetric rate of a turbulent flow, various empirical formulas are used, for example, an exponential one obtained after the substitution of the Blasius formula $\lambda = 0.3164\ /\mathrm{Re}^{0.25}$ into the Darcy–Weisbach equation to calculate the coefficient of hydraulic resistance; this formula is applied within the range $5 \cdot 10^3 < \mathrm{Re} < 5 \cdot 10^4$:

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$$Q_{ex} = 14.8 \left(\frac{\Delta P}{L\rho}\right)^{0.571} \left(\frac{R^{2.714}}{\nu^{0.143}}\right). \tag{2}$$

Since up to now there has been no efficient theory of a turbulent flow, this justifies the search for other analytical expressions for the derivation of which one could use certain model notions on the behavior of the particles of a flowing fluid.

Discussion of Results. Considering a fluid flow as a rolling motion of the spherical fragments of the medium (particles) of one layer over the surface of another one, adjacent to it and lying closer to the tube wall, we can obtain an equation for the distribution of velocities in a turbulent flow:

$$U = \frac{\nu}{\alpha} \ln\left(1 + 2\frac{y}{\alpha}\right), \tag{3}$$

where $\alpha = \nu/(u_* e)$ is the diameter of the fluid particles (rolling solids of revolution); $e = 2.72$ is the base of the natural logarithm.

Due to the presence of the forces of intermolecular interaction in a real fluid there are regions of ordering (associates, micelles, clusters, etc.), i.e., there exist short-range order. The dimensions of these kinetically individual supermolecular formations α depend on the chemical nature of the fluid and on the thermal motion intensity, as well as on the magnitude of the external loading applied. It should be noted that on increase in the shear stress (dynamic velocity), according to the formula $\alpha = \nu/(u_* e)$ the dimensions of particles become considerably smaller than the tube radius, $\alpha \ll R_w$.

An obvious merit of Eq. (3) is the definiteness and continuity of the obtained functional dependence $U = f(y)$ at all the points of the intratube space $0 \le y \le R_w$, as well as on the channel surface, when $y = 0$, with $U = 0$. Thus, the condition of "mechanical adherence" is described mathematically adequately. After integration with account for the fact that $S = \pi R^2$, $R = R_w - y$, and $dS = 2\pi(y - R_w)dy$, we obtain the following expression to calculate the volumetric rate of a turbulent flow in a cylindrical channel:

$$Q = \int_{R_w}^{0} U ds = \int_{R_w}^{0} \frac{\nu}{\alpha} \ln\left(1 + \frac{2}{\alpha} y\right) 2\pi (y - R_w)\, dy = \frac{2\pi\nu}{\alpha}\left[\frac{4R_w^2 + 4R_w\alpha + \alpha^2}{8} \ln\left(1 + 2\frac{R_w}{\alpha}\right) - \frac{R_w\alpha}{4} - \frac{3R_w^2}{4}\right].$$

Since the size of the particles of a low-viscosity fluid is much smaller than the channel radius ($\alpha \ll R_w$) and the numerical value $\ln 2e = \ln 5.44 \approx 3/2$, after some simplifications of the expression obtained we arrive at a formula that allows one to calculate the volumetric rate of a turbulent flow of a Newtonian fluid:

$$Q_{\log} = \pi R_w^2 u_* e \ln\left(\frac{R_w u_*}{\nu}\right). \tag{4}$$

Experimental verification of Eq. (4) was made on a turbulent rheometer [2]. Its main working element is an easily detachable pipe allowing one to carry out experiments at different geometric parameters (R_w and L) of the cylindrical channel. The upper open end of the pipe is in communication with the working chamber of the rheometer with a test fluid. Through the other end equipped with a cock the fluid flows out into the environment and thereafter into a receiving measuring cell. The pipe and the chamber are located in a thermostated jacket into which a heat carrier is supplied from a thermostat (cryostat). The rate of fluid displacement from the working chamber under the action of excess pressure ΔP is assigned and regulated with the aid of a gas system operating on nitrogen or an inert gas. The time interval t of the outflow of a fixed volume V of fluid is measured with the aid of an electronic stopwatch, after which the volumetric rate of fluid flow $Q = V/t$, shear stress, and the Reynolds number are calculated.

The physicochemical properties of the working media and the geometric parameters of capillaries are given in Table 1. The results of experimental verification of Eq. (4) for various fluids are given in Table 2, where Q_{exp} denotes the values of volumetric rates of flow obtained experimentally on the rheometer at different pressure drops (shear stresses), and Q_{ex} and $Q_{\log}$ are the values of volumetric flow rates calculated theoretically for the same pressure drops from Eqs. (2) and (4). It is apparent from the tables that the correspondence between Q_{exp} and $Q_{\log}$ is not worse than

TABLE 1. Values of Physicochemical Quantities of the Fluids Investigated and of the Geometric Characteristics of Cylindrical Channels

Fluid	ρ, kg/m^3	$\nu \cdot 10^6$, m^2/sec	L, m	$R_w \cdot 10^3$, m
Water	1000	1.0	1.45	2.05
Toluene	870	0.674	0.760	0.885
Heptane	684	0.613	0.838	0.845
Petroleum	820	3.0	0.760	0.885

TABLE 2. Hydrodynamic Parameters of a Flow, Experimental and Calculated Values of Volumetric Flow Rates of the Investigated Fluids

ΔP, Pa	τ_w, Pa	Re	$Q_{exp} \cdot 10^6$, m^3/sec	$Q_{ex} \cdot 10^6$, m^3/sec	$Q_{log} \cdot 10^6$, m^3/sec
		Water			
6890	4.87	3970	12.47	13.11	12.44
7840	5.54	4270	13.41	14.11	13.43
8800	6.22	4580	14.38	15.08	14.41
9750	6.89	4850	15.24	15.99	15.31
10710	7.57	5120	16.07	16.87	16.19
11671	8.25	5360	16.82	17.72	17.04
12620	8.92	5620	17.66	18.53	17.84
		Toluene			
19145	11.15	4100	3.84	3.98	3.78
25990	15.13	4860	4.55	4.74	4.55
57370	33.40	7670	7.18	7.44	7.27
102480	59.66	10880	10.19	10.37	10.23
201525	117.33	16200	15.15	15.25	15.16
297630	173.28	19950	18.71	19.05	19.01
397660	231.52	23750	22.24	22.48	22.49
493760	287.50	26920	25.21	25.44	25.48
		Heptane			
16250	8.2	4160	3.38	3.49	3.33
35800	18.1	6530	5.31	5.49	5.38
55500	28.0	8500	6.92	7.06	6.94
105000	52.9	12400	10.11	10.15	10.09
209000	105.4	18300	14.91	15.04	15.05
522200	263.3	31180	25.36	25.37	25.49
		Petroleum			
308000	179.33	3800	15.57	16.34	15.42
418000	243.38	4530	18.50	19.45	18.51
508000	295.78	5040	20.99	21.74	20.81

between Q_{exp} and Q_{ex}, which allows one to use Eq. (4) to calculate the magnitude of the volumetric rates of fluid flow in the studied range of Reynolds numbers 4000 < Re < 30,000.

Having divided the left- and right-hand sides of Eq. (4) by the value of the dynamic velocity u_*, we obtain

$$\frac{Q}{u_*} = A + B \ln u_* , \tag{5}$$

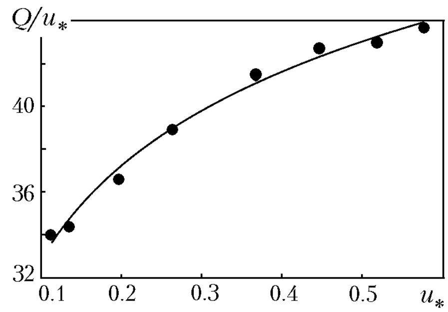

Fig. 1. Logarithmic dependence of the ratio between the volumetric and dynamic velocities on the dynamic velocity. $y = 6.333 \ln x + 47.425$, $R^2_{corr} = 0.9951$. u_*, m/sec; $Q/u_* \cdot 10^2$, m^2.

where $A = \pi R_w^2 e \ln \dfrac{R_w}{\nu}$ and $B = \pi R_w^2 e$ are constant values, when a fluid passes through a cylindrical channel with different shear stresses. The verification of Eq. (5) by using the experimental data obtained for a toluene flow (Table 2) has confirmed, with a high correlation factor (see Fig. 1), the logarithmic dependence of the ratio between the volumetric and dynamic velocities Q/u_* on the dynamic velocity (shear stress on the channel wall).

It is technically difficult to model a turbulent flow with higher Reynolds numbers under laboratory conditions. The verification of Eq. (4) carried out with the use of the operating data obtained at the Aleksandrovskoe–Anzhero-Sudzhensk trunk pipeline, through one of the sections of which ($L = 96 \cdot 10^3$ m, $R_w = 0.61$ m) petroleum of viscosity $\nu = 5.5 \cdot 10^{-6}$ m^2/sec and density $\rho = 830$ kg/m^3 was pumped [3] at a pressure drop $\Delta P = 6.5 \cdot 10^5$ Pa and Reynolds number Re $\approx$ 350,000, yielded the value of $Q_{log} = 1.65$ m^2/sec close to the experimentally controlled value $Q_{exp} = 1.68$ m^3/sec. The practical coincidence of the results for Q_{log} and Q_{exp} allows one to use Eq. (4) to predict the value of the volumetric flow rate in real pipelines and at rather high Reynolds numbers.

Conclusions. Assuming that the process of flow represents the rolling motion of spherical particles of one layer over the surface of the neighboring one and having statistically processed a multitude of experimental data on the behavior of working media during their turbulent flow in a cylindrical channel, we have suggested an empirical formula to calculate the volumetric rate of Newtonian fluid flow in the range of Reynolds numbers 4000 < Re < 30,000.

NOTATION

C, empirical coefficient; *L*, length of the tube, m; ΔP, pressure drop between the tube ends, Pa; *Q*, volumetric rate of flow of a fluid, m^3/sec; R_w, radius of the tube, m; Re, Reynolds number; R^2_{corr}, correlation factor; *S*, cross-sectional area of the tube, m^2; *t*, time of fluid outflow from a capillary, sec; *U*, linear velocity of a liquid, m/sec; u_*, dynamic velocity, m/sec; *V*, volume of a fluid, m^3; *x*, coordinate; *y*, distance from the considered layer to the tube wall, m; α, diameter of fluid particles, m; η, dynamic viscosity of a fluid, Pa·sec; λ, coefficient of hydrodynamic resistance; ρ, fluid density, kg/m^3; ν, kinematic viscosity of a fluid, m^2/sec; τ_w, shear stress, Pa; ξ, empirical coefficient. Subscripts: corr, correlation; ex, exponential; exp, experimental; log, logarithmic; w, wall.

REFERENCES

1. L. G. Loitsyanskii, *Mechanics of Liquids and Gases* [in Russian], Nauka, Moscow (1978).
2. A. Ya. Malkin, G. V. Nesyn, V. N. Manzhai, and A. V. Ilyushnikov, A new method of rheokinetic investigations based on the use of the Toms effect, *Vysokomolek. Soed. B*, **42**, No. 3, 377–384 (2000).
3. V. N. Manzhai, A. V. Ilyushnikov, M. M. Gareev, and G. V. Nesyn, Laboratory studies and commercial tests of a polymeric agent for reduction of the power consumption on an oil pipeline, *Inzh.-Fiz. Zh.*, **65**, No. 5, 515–517 (1993).